

Ginzburg-Landau Expansion and the Slope of the Upper Critical Field in Superconductors with Anisotropic Normal Impurity Scattering

A.I.Posazhennikova, M.V.Sadovskii
*Institute for Electrophysics,
 Russian Academy of Sciences, Ural Branch,
 Ekaterinburg, 620049, Russia
 E-mail: sadovski@ief.intec.ru*

Submitted to JETP, May 1997

Abstract

Ginzburg-Landau expansion for superconductors with anisotropic s - and d -wave pairing is derived in the presence of anisotropic normal impurity scattering which makes d -pairing state more stable under disordering. It is demonstrated that the slope of the upper critical field $|dH_{c2}/dT|_{T_c}$ in superconductors with d -wave pairing has nonlinear dependence on disorder, i.e. for the low anisotropic scattering rate it drops rather fast with concentration of normal impurities, but as anisotropy of scattering increases it features initial nonlinear growth, approaches a maximum and drops again, vanishing at the critical impurity concentration. In superconductors with anisotropic s -wave pairing $|dH_{c2}/dT|_{T_c}$ grows, approaching the known asymptotic behavior, characteristic of usual isotropic case irrespective to the scattering anisotropy.

PACS numbers: 74.20.Fg, 74.20.De

I. INTRODUCTION

The main problem of the present day physics of high-temperature superconductors is the determination of the nature and type of Cooper pairing. A majority of experiments and theoretical models [1] suggest the realization in these systems of anisotropic pairing with $d_{x^2-y^2}$ -symmetry with zeroes of the gap function at the Fermi surface. At the same time there exist some theoretical and experimental evidences [2,3] supporting the so-called anisotropic s -wave pairing. In this latest case there again appear zeroes (with no change of sign) or rather deep minima of the gap function in the same directions in the Brillouin zone as in the case of d -wave pairing. It was shown [4,5] that controlled disordering (introduction of normal impurities) can be an effective method of experimental discrimination between different types of anisotropic pairing. In our previous paper it was shown that one can use measurements of the slope of the upper critical field $|dH_{c2}/dT|_{T_c}$ for the same purpose, i.e. for the case of d -wave pairing the slope drops rather fast with disorder, while for the case of anisotropic s -wave pairing it grows with disorder as for the isotropic case.

Recently an interesting theoretical model was proposed [7] taking into account the effects of anisotropic (momentum-dependent) impurity scattering. It was shown that for large enough anisotropic " d -wave" scattering the usual pair-breaking effect of normal impurities (described by Abrikosov-Gorkov dependence for isotropic superconductor with magnetic impurities) is rather strongly suppressed. This allows, in principle at least, to overcome one of the main problems in the physics of high-temperature superconductors — the contradiction between the d -wave picture of pairing in these systems and their relative stability towards disordering [8]. There exist also some other explanations of such a stability (cf. an explanation proposed in our paper [9]), however the simplicity of the suggested model [7] is rather attractive and calls for further theoretical study of superconductors with exotic pairing with the account of anisotropic impurity scattering. The present paper is a natural continuation of our previous study, including the effects of anisotropic impurity scattering. It is demonstrated that anisotropic impurity scattering leads to significant anomalies in the dependence of the slope of the upper critical field on disorder (impurity concentration). As in the previous paper [6] our analysis is based on the microscopic derivation of Ginzburg-Landau expansion in impure system.

II. GINZBURG-LANDAU EXPANSION

Following Refs. [4,5], we analyze two-dimensional electronic system with isotropic Fermi surface and separable pairing potential of the form:

$$V(\mathbf{p}, \mathbf{p}') \equiv V(\phi, \phi') = -Ve(\phi)e(\phi'), \quad (1)$$

where ϕ is a polar angle, determining the electronic momentum direction in the conducting plane, and $e(\phi)$ is given by the following model dependence

$$e(\phi) = \begin{cases} \sqrt{2}\cos(2\phi) & (\text{d-wave}), \\ \sqrt{2}|\cos(2\phi)| & (\text{anisotropic s-wave}). \end{cases} \quad (2)$$

The pairing constant V is as usual different from zero in some region of the width of $2\omega_c$ around the Fermi level (ω_c -is some characteristic frequency of the quanta, responsible for

the pairing interaction). In this case the superconducting gap (order parameter) takes the form:

$$\Delta(\mathbf{p}) \equiv \Delta(\phi) = \Delta e(\phi), \quad (3)$$

and positions of its zeroes for s and d cases just coincide.

We examine a superconductor containing "normal" (nonmagnetic) impurities, which are chaotically distributed in space with concentration ρ . Following Ref. [7] we consider the square of the scattering amplitude of the impurity in the following form:

$$|V_{imp}(\mathbf{p}, \mathbf{p}')|^2 \equiv |V_{imp}(\phi, \phi')|^2 = |V_0|^2 + |V_1|^2 f(\phi) f(\phi'), \quad (4)$$

where V_0 is isotropic point-like scattering amplitude, V_1 is anisotropic scattering amplitude, and $f(\phi)$ is angular-dependent model function (ϕ is a polar angle mentioned above) which describes the type of anisotropic scattering. We consider the scattering to be essentially isotropic and impose the following constraints [7]:

$$|V_1|^2 \leq |V_0|^2; \quad \langle f \rangle = 0; \quad \langle f^2 \rangle = 1, \quad (5)$$

where $\langle \dots \rangle$ denotes the average value over the momentum direction on the Fermi surface (i.e. over the ϕ -angle). Accordingly, the second part in Eq.(4) represents the deviations from the isotropic scattering.

The normal and anomalous temperature Green's functions in such a superconductor are [10]:

$$G(\omega, \mathbf{p}) = -\frac{i\tilde{\omega} + \xi_{\mathbf{p}}}{\tilde{\omega}^2 + \xi_{\mathbf{p}}^2 + |\tilde{\Delta}(\mathbf{p})|^2}, \quad (6)$$

$$F(\omega, \mathbf{p}) = \frac{\tilde{\Delta}^*(\mathbf{p})}{\tilde{\omega}^2 + \xi_{\mathbf{p}}^2 + |\tilde{\Delta}(\mathbf{p})|^2},$$

where $\omega = (2n+1)\pi T$, ξ — is electronic energy with respect to the Fermi level,

$$\tilde{\omega}(\mathbf{p}) = \omega + i\rho \int \frac{d\mathbf{p}'}{(2\pi)^2} |V_{imp}(\mathbf{p} - \mathbf{p}')|^2 G(\omega, \mathbf{p}'), \quad (7)$$

$$\tilde{\Delta}(\mathbf{p}) = \Delta(\mathbf{p}) + \rho \int \frac{d\mathbf{p}'}{(2\pi)^2} |V_{imp}(\mathbf{p} - \mathbf{p}')|^2 F(\omega, \mathbf{p}').$$

To determine the transition temperature we can limit ourselves to Eqs.(7) linearized over Δ :

$$\tilde{\omega} = \omega + i\rho \frac{N(0)}{2\pi} \int d\xi \int_0^{2\pi} d\phi \left\{ |V_0|^2 + |V_1|^2 f(\phi) f(\phi') \right\} \frac{\tilde{\omega}}{\tilde{\omega}^2 + \xi^2}, \quad (8)$$

$$\tilde{\Delta} = \Delta + \rho \frac{N(0)}{2\pi} \int d\xi \int_0^{2\pi} d\phi \left\{ |V_0|^2 + |V_1|^2 f(\phi) f(\phi') \right\} \frac{\tilde{\Delta}}{\tilde{\omega}^2 + \xi^2}.$$

The critical temperature T_c is determined by the linearized gap-equation:

$$\Delta(\mathbf{p}) = -T_c \sum_{\omega} \int \frac{d\mathbf{p}'}{(2\pi)^2} V(\mathbf{p}, \mathbf{p}') \frac{\tilde{\Delta}(\mathbf{p}')}{\tilde{\omega}^2 + \xi_{\mathbf{p}'}^2} \quad (9)$$

Following standard procedure from Eqs.(8),(9) we obtain the following general equation for the critical temperature T_c :

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left(\langle e \rangle^2 + \langle ef \rangle^2 - 1\right) \left[\Psi\left(\frac{1}{2} + \frac{\gamma_0}{2\pi T_c}\right) - \Psi\left(\frac{1}{2}\right) \right] + \langle ef \rangle^2 \left[\Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{\gamma_0}{2\pi T_c} \left(1 - \frac{\gamma_1}{\gamma_0}\right)\right) \right] \quad (10)$$

where T_{c0} - is the transition temperature in the absence of impurities, $\Psi(x)$ - is the usual digamma function, $\gamma_0 = \pi\rho V_0^2 N(0)$ and $\gamma_1 = \pi\rho V_1^2 N(0)$ - correspondingly the isotropic and anisotropic impurity scattering rates, $\langle ef \rangle^2$ describes the “overlap” between the functions $e(\mathbf{p})$ and $f(\mathbf{p})$.

For the sake of simplicity we take the function $f(\mathbf{p})$ in the form analogous to that in Eq.(2):

$$f(\mathbf{p}) \equiv f(\phi) = \sqrt{2}\cos(2\phi), \quad (11)$$

This corresponds to the maximum overlap for d -case. More general treatment one could find in Ref. [7]. Now the renormalized Eqs.(8) can be written as follows:

$$\begin{aligned} \tilde{\omega} &= \omega + i\frac{\gamma_0}{\pi} \int d\xi \frac{\tilde{\omega}}{\tilde{\omega}^2 + \xi^2} + i\frac{\gamma_1}{\pi^2} \cos(2\phi) \int d\xi \int d\phi' \cos(2\phi') \frac{\tilde{\omega}}{\tilde{\omega}^2 + \xi^2}, \\ \tilde{\Delta} &= \Delta + i\frac{\gamma_0}{\pi} \int d\xi \frac{\tilde{\Delta}}{\tilde{\omega}^2 + \xi^2} + i\frac{\gamma_1}{\pi^2} \cos(2\phi) \int d\xi \int d\phi' \cos(2\phi') \frac{\tilde{\Delta}}{\tilde{\omega}^2 + \xi^2}. \end{aligned} \quad (12)$$

From here we obtain the well-known expression for the renormalized frequency in both cases:

$$\tilde{\omega} = \omega + \gamma_0 \text{sign}\omega. \quad (13)$$

In the case of d -wave pairing the gap symmetry in the presence of impurities is not changed:

$$\tilde{\Delta} = \Delta \frac{|\tilde{\omega}|}{|\tilde{\omega}| - \gamma_1}. \quad (14)$$

In the case of s -wave pairing the gap is shifted by a constant, which does not depend on ϕ and γ_1 :

$$\tilde{\Delta} = \Delta + \Delta_0 \frac{2\sqrt{2}\gamma_0}{\pi|\omega|}. \quad (15)$$

Finally T_c -equation for superconductor with d -wave pairing is written as:

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \left(1 - \frac{\gamma_1}{\gamma_0}\right) \frac{\gamma_0}{2\pi T_c}\right). \quad (16)$$

For superconductor with anisotropic s -wave pairing T_c -equation is written as:

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left(1 - \frac{8}{\pi^2}\right) \left[\Psi\left(\frac{1}{2} + \frac{\gamma_0}{2\pi T_c}\right) - \Psi\left(\frac{1}{2}\right) \right]. \quad (17)$$

Note that anisotropic scattering rate dependence drops from Eq.(17).

The appropriate dependencies of $T_c(\gamma_0/T_{c0})$ are shown in Fig.1, for the case of d -wave pairing with different values of the normalized anisotropic scattering rate γ_1/γ_0 . In the case of s -wave pairing the transition temperature T_c is slightly suppressed with the growth of γ_0/T_{c0} . In the case of d -wave pairing T_c suppression is rather fast for small values of γ_1 , but the critical value of γ_{0c}/T_{c0} leading to the complete destruction of superconducting state rapidly increases with the growth of the anisotropic scattering rate γ_1/γ_0 .

The gap function as usual can be used as an order parameter in the Ginzburg-Landau expansion for the free-energy density. We consider its amplitude $\Delta(T)$ to be a slowly varying function of the spatial coordinates. Accordingly in momentum space we get the Fourier-component of the order parameter:

$$\Delta(\phi, q) = \Delta_q(T)e(\phi). \quad (18)$$

The Ginzburg-Landau expansion for the free energy density difference between superconducting and normal state in the region of small q up to terms quadratic over Δ takes the form:

$$F_s - F_n = A|\Delta_q|^2 + q^2 C |\Delta_q|^2 \quad (19)$$

and is determined by the diagrams of loop-expansion for the free-energy of electrons moving in the field of superconducting order parameter fluctuations with some small vector q , shown in Fig.2. Diagrams (c) and (d) are to be subtracted, so that the coefficient A becomes zero at the transition point $T = T_c$.

Some details on calculating $\Gamma_{\mathbf{pp}'}$ and Ginzburg-Landau coefficients for the case of d -wave pairing can be found in the Appendices A and B. Note, that for the d -wave superconductors the contribution of diagrams Fig.2(b,d) actually vanishes up to terms of the order of q^2 , if we don't take into account an anisotropy of impurity scattering. In the case of s -wave superconductor all calculations are analogous, we only note that in such a case a dependence on anisotropic scattering rate is absent.

Finally we can express GL-coefficients in the following form:

$$A = A_0 K_A; \quad C = C_0 K_C, \quad (20)$$

where A_0 and C_0 are just the usual GL-coefficients for the case of isotropic s -wave pairing [11]:

$$A_0 = N(0) \frac{T - T_c}{T_c}; \quad C_0 = N(0) \frac{7\zeta(3)}{48\pi^2} \frac{v_F^2}{T_c^2}, \quad (21)$$

where $v_F, N(0)$ -are electron velocity and normal density of states at the Fermi level, and everything specific to our model is contained in dimensionless combinations K_A and K_C . In the absence of impurities for both models we obtain: $K_A^0 = 1$, $K_C^0 = 3/2$. For the impure system we get:

(A) d -wave pairing:

$$K_A = \frac{\gamma_0}{4\pi T_c} \int_{-\omega_c}^{\omega_c} \frac{d\xi}{\xi} \int_{-\infty}^{\infty} d\omega \frac{\omega + \xi}{(\omega^2 + \gamma_0^2) ch^2 \left(\frac{\omega + \xi}{2T_c} \right)} + \quad (22)$$

$$\frac{\gamma_1(2\gamma_0 + \gamma_1)}{4T_c} \int_{-\infty}^{\infty} d\omega \frac{\omega^2}{(\omega^2 + \gamma_0^2)(\omega^2 + (\gamma_0 - \gamma_1)^2)ch^2\left(\frac{\omega}{2T_c}\right)},$$

$$K_C = \frac{3\pi T_c}{7\zeta(3)\gamma_1} \left\{ \frac{2\pi T_c}{\gamma_1} \left[\Psi\left(\frac{1}{2} + \frac{\gamma_0 - \gamma_1}{2\pi T_c}\right) - \Psi\left(\frac{1}{2} + \frac{\gamma_0}{2\pi T_c}\right) \right] + \Psi'\left(\frac{1}{2} + \frac{\gamma_0 - \gamma_1}{2\pi T_c}\right) \right\}; \quad (23)$$

(B) anisotropic s -wave pairing:

$$K_A = \frac{\gamma_0}{\pi T_c} \left\{ \frac{1}{4} \int_{-\omega_c}^{\omega_c} \frac{d\xi}{\xi} \int_{-\infty}^{\infty} d\omega \frac{\omega + \xi}{(\omega^2 + \gamma_0^2)ch^2\left(\frac{\omega + \xi}{2T_c}\right)} + \frac{2\gamma_0}{\pi} \int_{-\infty}^{\infty} d\omega \frac{1}{(\omega^2 + \gamma_0^2)ch^2\left(\frac{\omega}{2T_c}\right)} \right\}, \quad (24)$$

$$K_C = -\frac{3(\pi^2 - 8)}{28\pi^2\zeta(3)} \Psi''\left(\frac{1}{2} + \frac{\gamma_0}{2\pi T_c}\right) + \frac{24\pi^2}{7\zeta(3)\gamma_0^2} \frac{T_c^2}{(\pi^2 - 8)\gamma^2} \ln\left(\frac{T_c}{T_{c0}}\right) + \frac{6\pi}{7\zeta(3)} \frac{T_c}{\gamma_0}. \quad (25)$$

The appropriate dependencies of dimensionless coefficients on disorder parameter γ_0/T_{c0} in the case of d -wave pairing and for different values of the normalized anisotropic scattering rate γ_1/γ_0 are shown in Figs.3,4.

III. UPPER CRITICAL FIELD

GL-coefficients A and C , as usual, define the temperature dependence of the upper critical magnetic field close to T_c : [11]:

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2(T)} = -\frac{\phi_0}{2\pi} \frac{A}{C} \quad (26)$$

where $\phi_0 = c\pi/e$ – is magnetic flux quantum, $\xi(T)$ – is temperature dependent coherence length. Now we can easily find the “slope” of the temperature dependence of $H_{c2}(T)$ near T_c , i.e. the temperature derivative:

$$\left| \frac{dH_{c2}}{dT} \right|_{T_c} = \frac{24\pi\phi_0}{7\zeta(3)v_F^2} T_c \frac{K_A}{K_C} \quad (27)$$

In the case of the usual s -wave superconductivity anisotropic scattering does not influence the behavior of the slope of the upper critical field. The appropriate dependencies of dimensionless parameter $h = |dH_{c2}/dT|_{T_c}/|dH_{c2}/dT|_{T_{c0}}$ on disorder γ_0/T_{c0} in the case of d -wave pairing – for different values of the normalized anisotropic scattering rate γ_1/γ_0 are shown in Fig.5. In the case of anisotropic s -wave pairing the slope as usual [6] grows with disorder and in the limit of strong disorder $\gamma_0 \gg T_c$ it crosses over to the usual linear dependence described by the well-known Gorkov’s expression [12]:

$$\frac{\sigma}{N(0)} \left| \frac{dH_{c2}}{dT} \right|_{T_c} = \frac{8e^2}{\pi^2} \phi_0 \quad (28)$$

where $\sigma = N(0)e^2v_F^2/3\gamma_0$ – is electron conductivity in the normal state, which is characteristic of the impure superconductors with isotropic s -wave pairing. It means that strong

disordering suppresses gap anisotropy and we obtain a usual limit of the impure superconductor.

For the case of d -wave pairing the slope of H_{c2} drops to zero on the scale $\gamma_0 \sim T_{c0}$ for the small values of the rate γ_1/γ_0 . For the values of anisotropic scattering rate on the interval $0.5 \leq \gamma_1/\gamma_0 \leq 0.6$ the behavior of the slope is qualitatively changed: h smoothly but nonlinearly increases with the growth of γ_0/T_{c0} , crosses over the maximum and then has a sharp drop. The interval where the slope grows extends as γ_1 approaches γ_0 . In our opinion these sharp anomalies in dependence of the slope of the upper critical field on disorder can be used for determining the pairing type and possible role of anisotropic impurity scattering in "unusual" superconductors. Unfortunately, in case of high- T_c oxides the situation is complicated by the known nonlinearity of temperature dependence of H_{c2} , which is observed in rather wide region close to T_c and also by some ambiguity in experimental methods to measure H_{c2} .

This work was partly supported by the grant No.96-02-16065 of the Russian Foundation of Fundamental Research. It was performed under the project IX.1 of the State Program "Statistical physics" as well as the project No.96-051 of the State Program on HTSC of the Russian Ministry of Science.

APPENDIX A: VERTEX PART $\Gamma_{\mathbf{p}\mathbf{p}'}$ IN “LADDER” APPROXIMATION.

The Bethe-Salpeter equation for the vertex part takes the form:

$$\Gamma_{\mathbf{p}\mathbf{p}'} = U(\mathbf{p}, \mathbf{p}') + \sum_{\mathbf{p}''} U(\mathbf{p}, \mathbf{p}'') G^R(\mathbf{p}'') G^A(\mathbf{p}'') \Gamma_{\mathbf{p}''\mathbf{p}'}, \quad (\text{A1})$$

where $U(\mathbf{p}, \mathbf{p}')$ -is irreducible vertex function. We take $U(\mathbf{p}, \mathbf{p}')$ in the following form (“ladder” approximation):

$$U(\mathbf{p}, \mathbf{p}') = \rho V_0^2 + \rho V_1^2 f(\mathbf{p}) f(\mathbf{p}'). \quad (\text{A2})$$

Then Eq.(A1) can be written as:

$$\Gamma_{\mathbf{p}\mathbf{p}'} = \rho V_0^2 + \rho V_1^2 f(\mathbf{p}) f(\mathbf{p}') + \rho V_0^2 \Psi(\mathbf{p}') + \rho V_1^2 f(\mathbf{p}) \Phi(\mathbf{p}') \quad (\text{A3})$$

where

$$\begin{aligned} \Psi(\mathbf{p}') &= \sum_{\mathbf{p}''} G^R(\mathbf{p}'') G^A(\mathbf{p}'') \Gamma_{\mathbf{p}''\mathbf{p}'}, \\ \Phi(\mathbf{p}') &= \sum_{\mathbf{p}''} f(\mathbf{p}'') G^R(\mathbf{p}'') G^A(\mathbf{p}'') \Gamma_{\mathbf{p}''\mathbf{p}'}. \end{aligned} \quad (\text{A4})$$

From Eq.(A3) one can obtain a self-consistent set of equations for $\Psi(\mathbf{p}')$ and $\Phi(\mathbf{p}')$:

$$\begin{cases} \Psi(\mathbf{p}') = \rho V_0^2 I_1 + \rho V_1^2 f(\mathbf{p}') I_2 + \rho V_0^2 I_1 \Psi(\mathbf{p}') + \rho V_1^2 I_2 \Phi(\mathbf{p}'), \\ \Phi(\mathbf{p}') = \rho V_0^2 I_2 + \rho V_1^2 f(\mathbf{p}') I_3 + \rho V_0^2 I_2 \Psi(\mathbf{p}') + \rho V_1^2 I_3 \Phi(\mathbf{p}'), \end{cases} \quad (\text{A5})$$

where

$$\begin{aligned} I_1 &= \sum_{\mathbf{p}} G^R(\mathbf{p}) G^A(\mathbf{p}), \\ I_2 &= \sum_{\mathbf{p}} f(\mathbf{p}) G^R(\mathbf{p}) G^A(\mathbf{p}), \\ I_3 &= \sum_{\mathbf{p}} f^2(\mathbf{p}) G^R(\mathbf{p}) G^A(\mathbf{p}). \end{aligned} \quad (\text{A6})$$

Solving system (A5), one can find the appropriate expressions for $\Psi(\mathbf{p}')$ and $\Phi(\mathbf{p}')$ and hence the expression for the vertex part:

$$\Gamma_{\mathbf{p}\mathbf{p}'} = \frac{\rho V_0^2 (1 - \rho V_1^2 I_3 + \rho V_1^2 f(\mathbf{p}') I_2) + \rho V_1^2 (f(\mathbf{p}) f(\mathbf{p}') (1 - \rho V_0^2 I_1) + \rho V_0^2 f(\mathbf{p}) I_2)}{(1 - \rho V_0^2 I_1)(1 - \rho V_1^2 I_3) - \rho V_0^2 \rho V_1^2 I_2^2} \quad (\text{A7})$$

APPENDIX B: GINZBURG-LANDAU COEFFICIENTS.

We can easily see that the contribution of the diagram Fig.2(a) is

$$\begin{aligned}
& -\frac{T}{(2\pi)^2}\Delta_q^2\sum_{\omega}\int d\mathbf{p}2\cos^2(2\phi)G_{\omega}(\mathbf{p}_+)G_{-\omega}(\mathbf{p}_-)= \\
& -\Delta_q^2TN(0)\sum_{\omega}\int\frac{d\xi}{\tilde{\omega}^2+\xi^2}+\Delta_q^2q^2\frac{N(0)\pi v_F^2T_c}{8}\sum_{\omega}\frac{1}{|\tilde{\omega}|^3}.
\end{aligned} \tag{B1}$$

The contribution of the diagram Fig.2(b) is

$$-\frac{T}{(2\pi)^2}\Delta_q^2\sum_{\omega}\int d\mathbf{p}2\cos^2(2\phi)G_{\omega}(\mathbf{p})G_{-\omega}(\mathbf{p})=-\Delta_q^2T_cN(0)\sum_{\omega}\int\frac{d\xi}{\tilde{\omega}^2+\xi^2}. \tag{B2}$$

The contribution of the diagram with diffusion propagator Fig.2(b) is

$$-T\sum_{\omega}\sum_{\mathbf{p}\mathbf{p}'}\sqrt{2}\cos(2\phi)G^R(\mathbf{p}_+)G^A(\mathbf{p}_-)\Gamma_{\mathbf{p}\mathbf{p}'}\sqrt{2}\cos(2\phi')G^R(\mathbf{p}'_+)G^A(\mathbf{p}'_-). \tag{B3}$$

Taking into account (A6) and (A7) we get from here

$$-TN(0)\pi\gamma_1\sum_{\omega}\left[\frac{1}{|\tilde{\omega}|(|\tilde{\omega}|-\gamma_1)}-\frac{v_F^2(2|\tilde{\omega}|-\gamma_1)q^2}{8|\tilde{\omega}|^3(|\tilde{\omega}|-\gamma_1)^2}\right]. \tag{B4}$$

Note that in the absence of anisotropic scattering for the case of d -pairing the contribution of diagrams (c) actually vanishes up to terms of the order of q^2 .

In the same way we get the appropriate contribution of the diagram (d)

$$-TN(0)\pi\gamma_1\sum_{\omega}\frac{1}{|\tilde{\omega}|(|\tilde{\omega}|-\gamma_1)}. \tag{B5}$$

Finally we get the expression for $F_s - F_n$ and so Ginzburg-Landau coefficients cited in the main body of the paper.

FIGURE CAPTIONS

Fig.1. Critical temperature T_c as a function of the normalized isotropic scattering rate γ_0/T_{c0} . The dashed curve represents the s -wave pairing case, the solid curves represent the d -wave pairing case for different values of the normalized anisotropic scattering rate γ_1/γ_0 :

1— $\gamma_1/\gamma_0=0.0$; 2—0.3; 3—0.5; 4—0.6; 5—0.7; 6—0.8; 7—0.9; 8—0.95.

Fig.2. Diagrammatic representation of Ginzburg-Landau expansion. Electronic lines are "dressed" by impurity scattering. Γ is the impurity vertex calculated in "ladder" approximation. Diagrams (c) and (d) are calculated with $q=0$ and $T=T_c$.

Fig.3. Dependence of dimensionless coefficient K_A/K_{A0} on disorder parameter γ_0/T_{c0} . The dashed curve represents the s -wave pairing case, the solid curves represent the d -wave pairing case for different values of the normalized anisotropic scattering rate γ_1/γ_0 :

1— $\gamma_1/\gamma_0=0.0$; 2—0.4; 3—0.6; 4—0.7; 5—0.8; 6—0.9; 7—0.95.

Fig.4. Dependence of dimensionless coefficient K_C/K_{C0} on disorder parameter γ_0/T_{c0} . The dashed curve represents the s -wave pairing case, the solid curves represent the d -wave pairing case for different values of the normalized anisotropic scattering rate γ_1/γ_0 :

1— $\gamma_1/\gamma_0=0.0$; 2—0.4; 3—0.6; 4—0.7; 5—0.8; 6—0.9; 7—0.95.

Fig.5. Dependence of normalized slope of the upper critical field $h = \left| \frac{dH_{c2}}{dT} \right|_{T_c} / \left| \frac{dH_{c2}}{dT} \right|_{T_{c0}}$ on disorder parameter γ_0/T_{c0} . The dashed curve represents the s -wave pairing case, the solid curves represent the d -wave pairing case for different values of the normalized anisotropic scattering rate γ_1/γ_0 :

1— $\gamma_1/\gamma_0=0.0$; 2—0.4; 3—0.5; 4—0.6; 5—0.7; 6—0.8; 7—0.9; 8—0.95.

REFERENCES

- [1] Pines D. Physica **C235-240**, 113 (1994).
- [2] Chakravarty S., Subdø A., Anderson P.W., Strong S. Science **261**, 337 (1993).
- [3] Liechtenstein A.I., Mazin I.I., Andersen O.K. Phys.Rev.Lett. **74**, 2303 (1995).
- [4] Borkovski L.S., Hirschfeld P.J. Phys.Rev. **B49**, 15404 (1994).
- [5] Fehrenbacher R., Norman M.R. Phys.Rev. **B50**, 3495 (1994).
- [6] Posazhennikova A.I., Sadovskii M.V. Pis'ma ZhETF **63**, 347 (1996);
JETP Lett., **63**, 358 (1996).
- [7] Haran G., Nagi A.D.S. Phys.Rev. **54**, 15463 (1996).
- [8] Sadovskii M.V. Physics Reports **282**, 225 (1997)
- [9] Sadovskii M.V., Posazhennikova A.I. Pis'ma ZhETF **65**, 258 (1997);
JETP Lett., **65**, 270, (1997)
- [10] Abrikosov A.A., Gorkov L.P., Dzyaloshinskii I.E. Methods of Quantum Field Theory in Statistical Physics. Pergamon Press, Oxford 1965.
- [11] De Gennes P.G. Superconductivity of Metals and Alloys. W.A.Benjamin, N.Y. 1966.
- [12] Gorkov L.P. Zh.Eksp.Teor.Phys.(JETP) **37**, 1407 (1959).

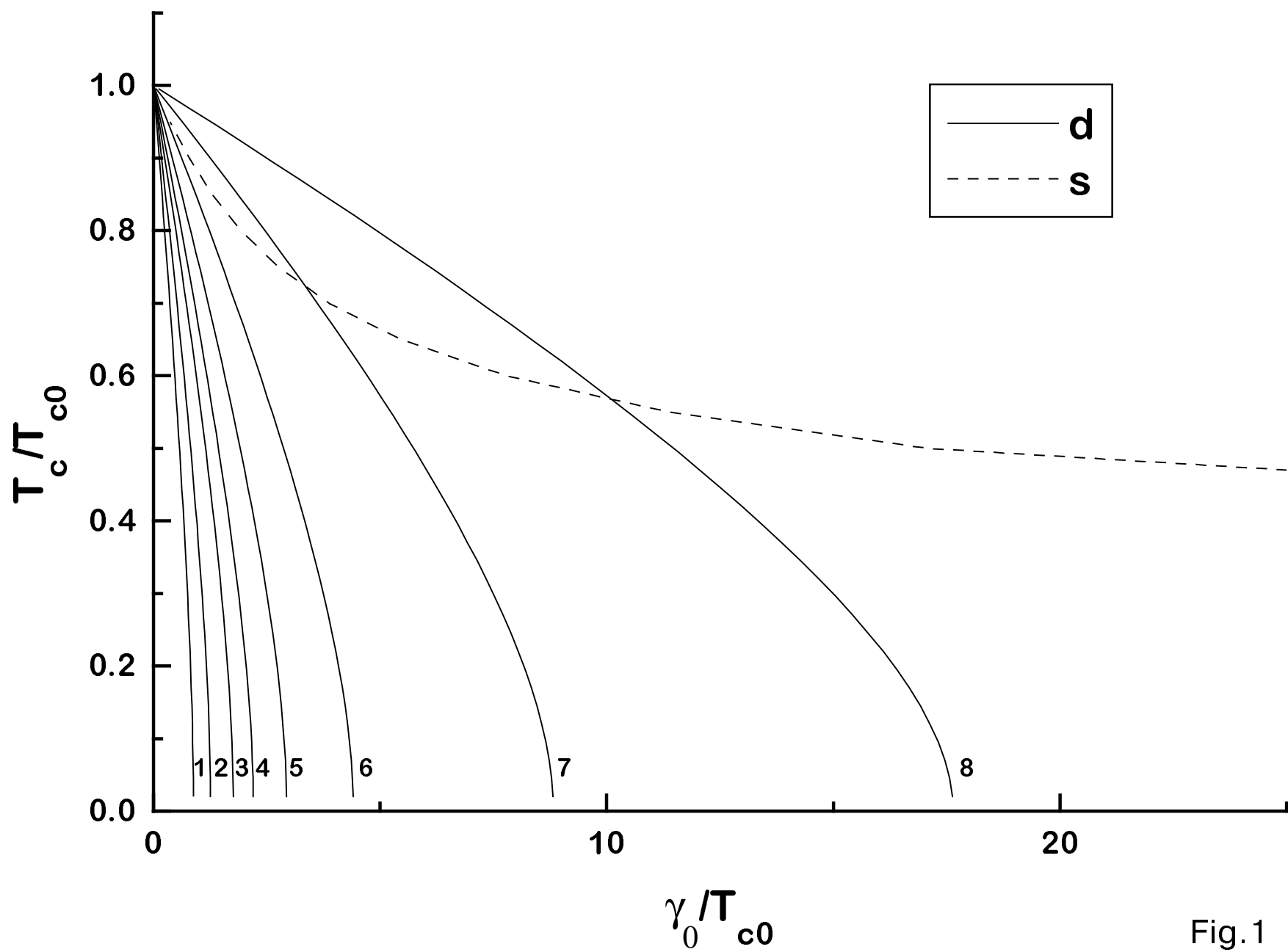


Fig.1

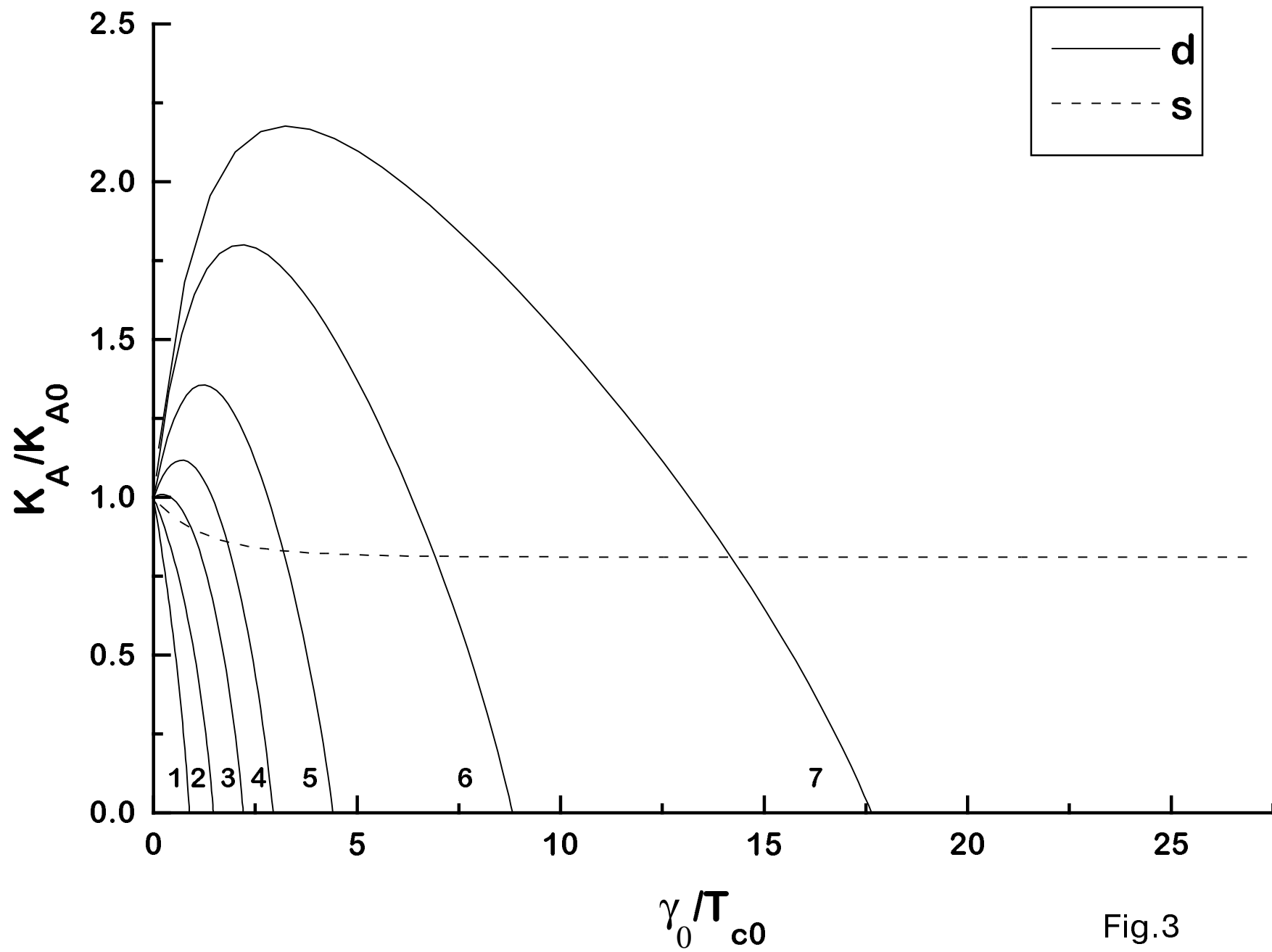


Fig.3

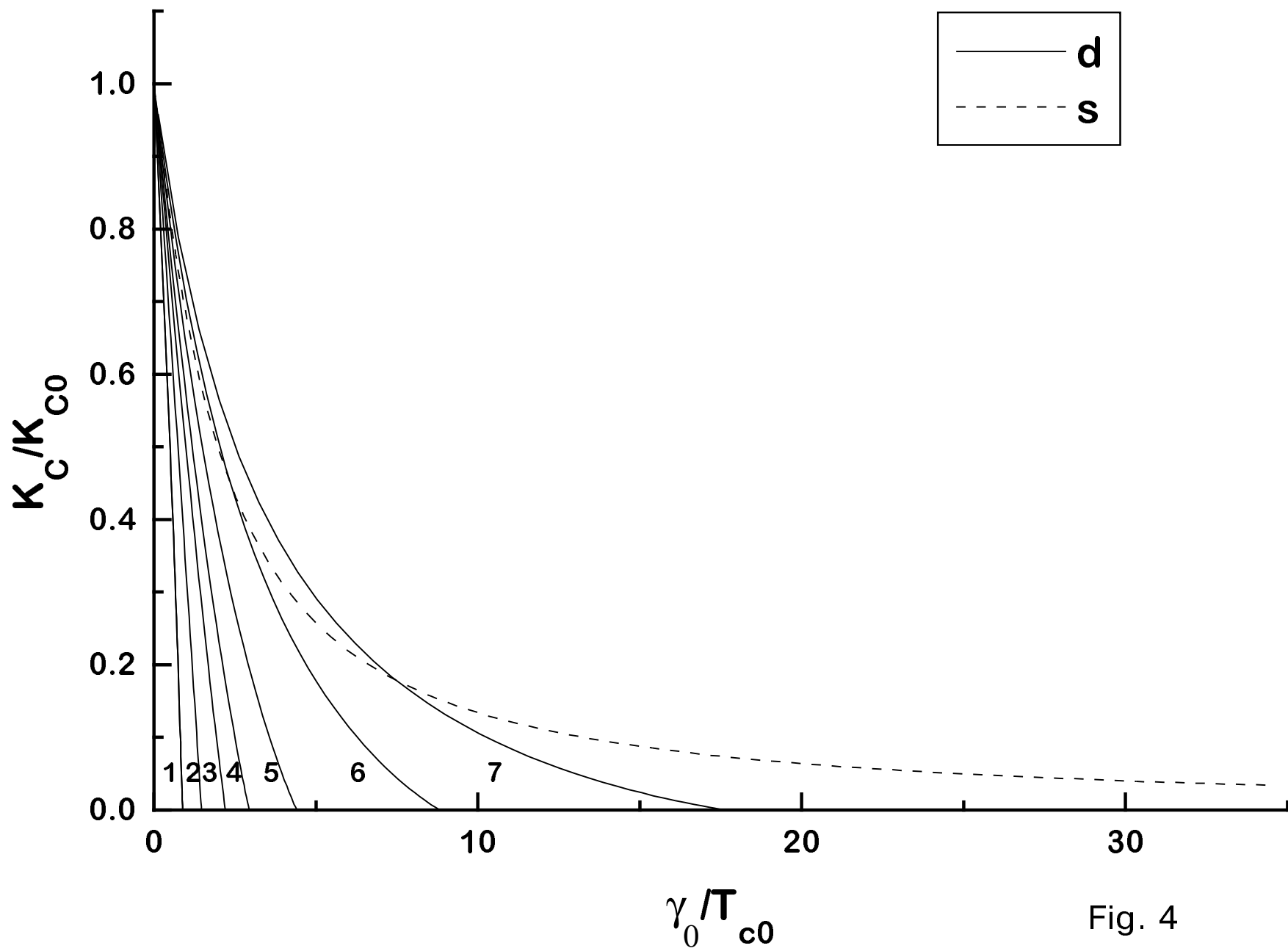


Fig. 4

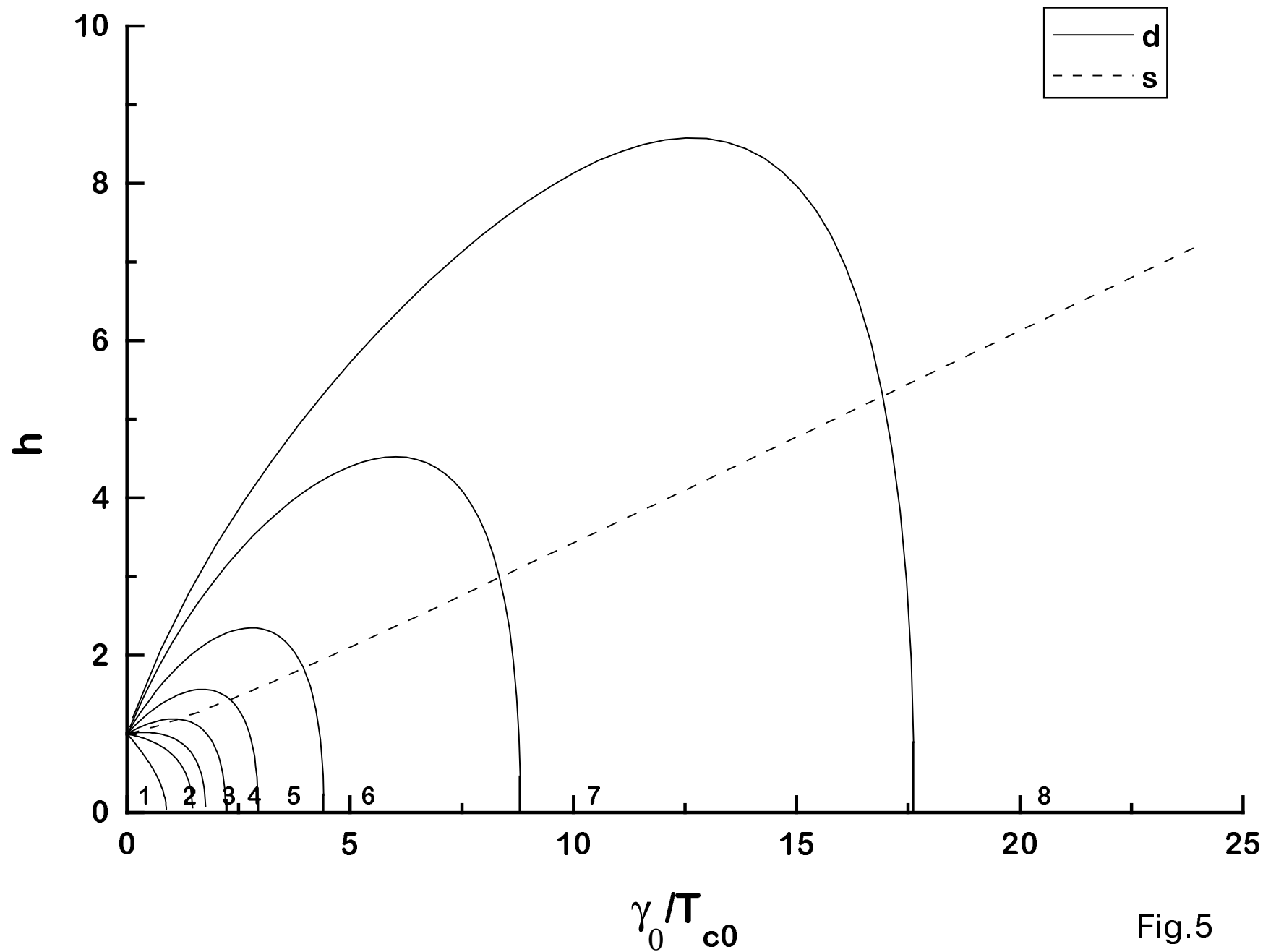


Fig.5

